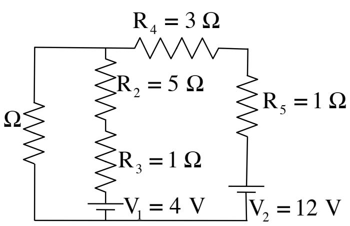
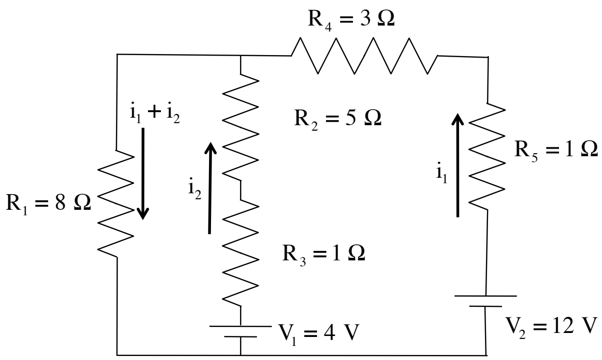
Problem 28.21

The circuit is run for 120 seconds.

a.) What's the current in each resistor?

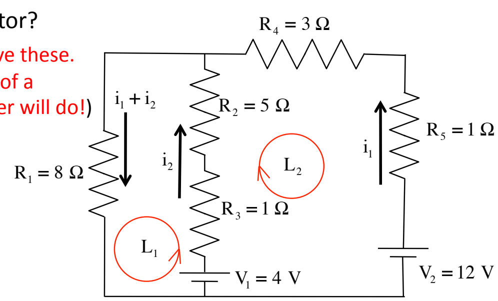


Below is an enlarged circuit with currents defined on a best-guess basis (more about current definitions at the end of the problem):



a.) What's the current in each resistor?

(Note that there are several ways to solve these. I've used an algebraic approach instead of a matrix-driven calculator approach. Either will do!)



The *node equation* was used to define the current through the 8 ohm resistor, so with only two unknowns, we need to use *loop equations* to finish the task. I've identified the loops and direction of transverse on the sketch.

L_1 :

$$V_1 - i_2R_3 - i_2R_2 - (i_1 + i_2)R_1 = 0$$
$$4 - i_2(1) - i_2(5) - i_1(8) - i_2(8) = 0$$
$$\Rightarrow\ 8i_1 + i_2(14) = 4$$
$$\Rightarrow\ i_1 = \frac{-14i_2 + 4}{8}$$
$$\Rightarrow\ i_1 = -1.75i_2 + .5$$

L_2 :

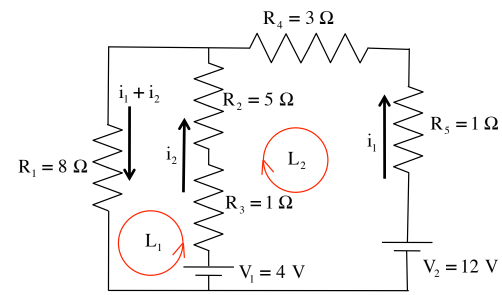
$$V_1 - i_2R_3 - i_2R_2 + i_1R_4 + i_1R_5 - V_2 = 0$$
$$4 - i_2(1) - i_2(5) + i_1(3) + i_1(1) - 12 = 0$$
$$\Rightarrow\ -i_2(6) + i_1(4) = 8$$
$$\Rightarrow\ 4i_1 - 6i_2 = 8$$
$$\Rightarrow\ 4(-1.75i_2 + .5) - 6i_2 = 8$$
$$\Rightarrow\ -7i_2 + 2 - 6i_2 = 8$$
$$\Rightarrow\ i_2 = -.46$$

So:

$$i_1 = -1.75i_2 + .5$$
$$= -1.75(-.46) + .5$$
$$= 1.3\ A$$

and the current through the 8 ohm resistor is:

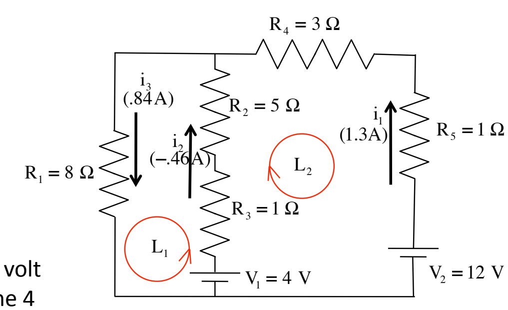
$$i_3 = i_1 + i_2$$
$$= 1.3 + (-.46)$$
$$= .84\ A$$



Note that the negative sign for i_2 means that the current is really in the opposite direction as defined. This is why assumptions about current direction is not important. Apparently, the 12 volts battery is large enough to stifle the effects of the 4 volt battery making the current through that branch opposite the direction you would expect if the 4 volt battery stood alone. When a "bad" assumption is made, though, nothing is hurt. It just means that when you solve for that variable, you will find a negative sign in front of its numeric value. No bid deal!

b.) The energy delivered by each battery makes use of the power relationship. For V_1 :

$$P_{V_1} = i_2V_1$$
$$= (-.46\ A)(4\ V)$$
$$= -1.84\ W$$

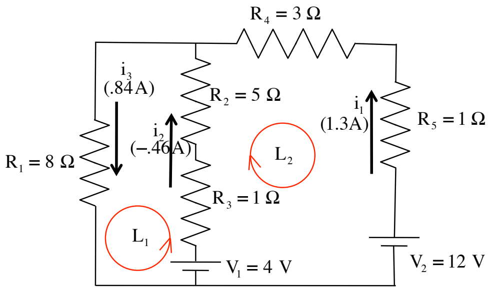


Why the negative sign? The 12 volt battery is forcing current *into* the 4 volt battery. This is opposite what battery usually do (they are designed to put energy into the system). As such, the battery in this case can be thought of as depleting energy, suggests a negative power rating.

Knowing the power, we can write:

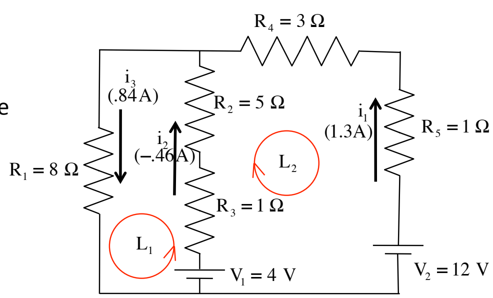
$$P_{V_1} = \frac{W_{battery1}}{\Delta t}$$
$$\Rightarrow\ W_{battery1} = P\Delta t$$
$$= \left(-1.84\ \text{joules}/\text{sec}\right)(120\ \text{sec})$$
$$= -222\ \text{joules}$$

b.) The energy delivered by each battery makes use of the power relationship. For V_2 :



$$P_{V_2} = i_1 V_2$$
$$= (1.3\ \text{A})(12\ \text{V})$$
$$= 15.6\ \text{Watts} = \frac{W_{battery 2}}{\Delta t}$$
$$\Rightarrow\quad W_{battery 2} = P_{V_2}\Delta t$$
$$= \left(15.6\ \text{joules}/\text{sec}\right)(120\ \text{sec})$$
$$= 1872\ \text{joules}$$

c.) The energy delivered to the resistors is the same as the energy dissipated by them, so using the power relationship for resistors, we can write:

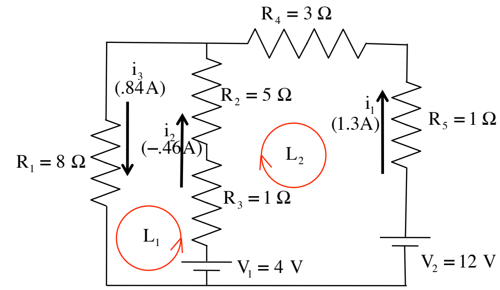


$$P_{R_1} = i_3^2 R_1$$
$$= (.84\ \text{A})^2 (8\ \Omega)$$
$$= 5.65\ \text{joules}/\text{sec}$$
$$\text{As } P_{R_1} = \frac{W_{resistor 1}}{\Delta t}$$
$$\Rightarrow\quad W_{resistor 1} = P_{R_1}\Delta t$$
$$= \left(5.6\ \text{joules}/\text{sec}\right)(120\ \text{sec})$$
$$= 672\ \text{joules}$$

Note that the actual solution manual lists this as 687 joules because they used .846 amps as their current.

c.) (cont'd)

For R_2:

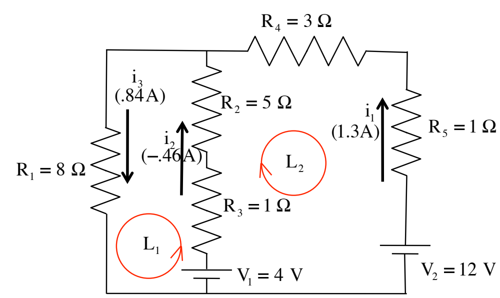


$$P_{R_2} = i_2^2 R_2$$
$$= (-.46\ \text{A})^2 (5\ \Omega)$$
$$= 1.06\ \text{joules}/\text{sec}$$
$$\text{As } P_{R_2} = \frac{W_{resistor 2}}{\Delta t}$$
$$\Rightarrow\quad W_{resistor 2} = P_{R_2}\Delta t$$
$$= \left(1.06\ \text{joules}/\text{sec}\right)(120\ \text{sec})$$
$$= 127\ \text{joules}$$

Note that the actual solution manual lists this as 128 joules as they use -.462 amps.

c.) (cont'd)

For R_3:



$$P_{R_3} = i_2^2 R_3$$
$$= (-.46\ \text{A})^2 (1\ \Omega)$$
$$= .21\ \text{joules}/\text{sec}$$
$$\text{As } P_{R_3} = \frac{W_{resistor 3}}{\Delta t}$$
$$\Rightarrow\quad W_{resistor 3} = P_{R_3}\Delta t$$
$$= \left(.21\ \text{joules}/\text{sec}\right)(120\ \text{sec})$$
$$= 25.2\ \text{joules}$$

Note that the actual solution manual lists this as 25.6 joules.

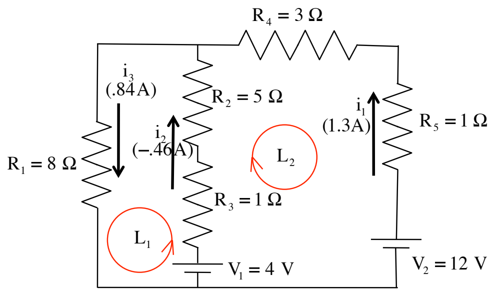
c.) (cont'd)

For R_4:

$$P_{R_4} = i_1^2 R_4$$
$$= (1.3\ \text{A})^2 (3\ \Omega)$$
$$= 5.07\ \text{joules}/\text{sec}$$

As $P_{R_4} = \dfrac{W_{\text{resistor}4}}{\Delta t}$

$$\Rightarrow W_{\text{resistor}4} = P_{R_4}\Delta t$$
$$= \left(5.07\ \text{joules}/\text{sec}\right)(120\ \text{sec})$$
$$= 609\ \text{joules}$$



Note that the actual solution manual lists this as 616 joules as they use 1.31 amps.

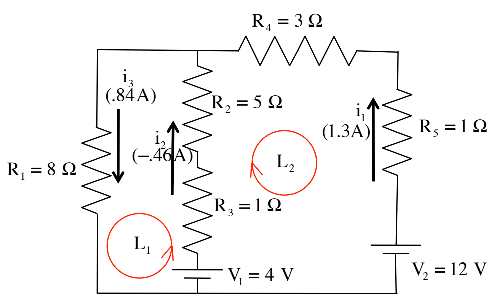
c.) (cont'd)

For R_5:

$$P_{R_5} = i_1^2 R_5$$
$$= (1.3\ \text{A})^2 (\ \Omega)$$
$$= 1.69\ \text{joules}/\text{sec}$$

As $P_{R_5} = \dfrac{W_{\text{resistor}5}}{\Delta t}$

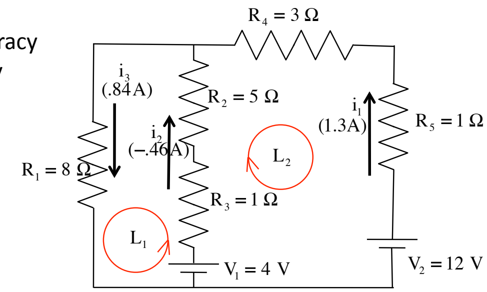
$$\Rightarrow W_{\text{resistor}5} = P_{R_5}\Delta t$$
$$= \left(1.69\ \text{joules}/\text{sec}\right)(120\ \text{sec})$$
$$= 203\ \text{joules}$$



Note that the actual solution manual lists this as 205 joules.

d.) Using the work/energy values provided by the book (for accuracy sake), the sum of all the energy dissipated by the resistors is:

$$W_{\text{donebyresistors}} = 687 + 128 + 25.6 +$$
$$+ 616 + 205 = 1661.6\ \text{joules}$$



The chemical energy conversion in battery V_1 provided 1880 joules (according to the book). So what happened to the approximately 220 joules of "left-over" energy provided by that battery? It went into charging up battery V_2 to the tune of -222 joules (this is the approximate overlay—the disparity is due to round-off error).

e.) The total energy dissipated by the resistors is, as stated above, 1661.6 joules (approximately). This matches (again, as was stated above) the amount of energy (net) put into the system by the batteries.